Studia Philosophiae Christianae
UKSW
56(2020) Special Issue 2

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MATHEMATICALNESS OR MATHEMATICABILITY OF NATURE?*

Abstract. The notions of "mathematicalness" and "mathematicability" of nature appear in the context of attempts at explaining the effectiveness of mathematics in the description of the world. Mathematicalness of nature means that structures of the world are mathematical. But is this true? Is nature mathematical? In the paper some reasons for mathematicalness of nature are considered. Mathematical analysis is widely used in physics. Its application requires continuity of time and space. There are also different kinds of infinity in the mathematical theories used in physics. This raises the issue: whether the material world is continuous or we "impose" on nature certain properties in order to use convenient mathematical tools. Is mathematics a useful tool, or does it reflect nature? So, is nature mathematical or only mathematicable? The article shows that mathematicalness of nature is only a metaphysical hypothesis.

Keywords: nature; mathematics; science; mathematicalness of nature; mathematicability of nature

1. INTRODUCTION

It is a truism to say that mathematics is successfully used in natural sciences, especially in physics whose theories are generally similar to mathematical theories to the extent that today it is difficult to perceive the boundary where mathematical formalism ends and physics,

* This article was originally published in Polish as: A. Lemańska, *Matematyczność czy matematyzowalność przyrody?*, Studia Philosophiae Christianae 49(2013)3, 5-24. The translation of the article into English was financed by the Ministry of Science and Higher Education of the Republic of Poland as part of the activities promoting science – Decision No. 676/P-DUN/2019 of 2 April 2019. Translation made by GROY Translations.

understood as the description of natural phenomena, begins. Also, other natural sciences, although they are not mathematicalised to the same extent as physics, use a variety of mathematical models and theories. In this sense it can also be said that nature is mathematicable, which means that it has properties enabling the application of mathematical formalism in the theories of natural sciences.

The explanation of matematicability of nature is not a trivial issue. This is because we are dealing with the physical, material world, the spatial and temporal reality on the one hand, and with mathematical objects which, apart from their essence, are certainly not material objects immersed in time and space. Why therefore the science about such objects – mathematics – is used for describing and explaining the world of physical objects whose nature is different? There are numerous answers to this question and, as it can easily be seen, they significantly depend on the interpretation of both the essence of mathematics, and the relationship between the theory of nature and the material world, which leads to the domain of controversy in the philosophy of mathematics and the philosophy of science. In this article, I will not dispute with different views in this scope. I will only have a closer look at one issue which arises in the context of the question about the matematicability of nature. Namely, in some explanations of the effectiveness of mathematics in the research on nature, a hypothesis about the "mathematicalness of nature" appears, by which the existence of correspondence between mathematical and natural structures is meant. If nature is mathematical, then explaining the fact of the "unreasonable effectiveness of mathematics"[1] becomes a trivial task. But – is nature mathematical in its essence? In this article I will point out certain difficulties with accepting a positive answer to this question.

2. THE CONCEPT OF THE "MATHEMATICALNESS OF NATURE"

In literature, there are more than one interpretation of the concept of "mathematicalness of nature". Most authors analysing the rela-

1 P. Wigner, *The Unreasonable Effectiveness of Mathematics in the Natural Sciences*, Communications in Pure and Applied Mathematics 13(1960)1, 1–14.

tions between mathematics and the material world, form their own descriptions of this term, which often depend on their scientific discipline. We are therefore dealing with an entire palette of positions, from moderate ones which almost reduce the mathematicalness of nature to its mathematicability, to the most extreme ones which connect the mathematicalness of nature with mathematical platonism[2]. The common core of these different concepts is the belief that mathematicalness is a feature of physical reality which consists in the fact that, as Józef Życiński writes, "there is a puzzling correspondence between natural phenomena and their mathematical description, which is in no way limited to the generalisations of registered observations, but it contains a surplus of information"[3]. This is why "the world is so willing to succumb to mathematised research"[4]. Therefore the question whether or not nature is mathematical, comes down to determining whether correspondence between natural and mathematical objects exists and in what it would consist. Małgorzata Czarnocka lists the following interpretations of the position assuming that nature is mathematical (these interpretations presuppose epistemological realism): "as a similarity of the mathematical and natural universes in question or their subdomains, as gen-identity (universes would be identical but not the same) or quasi-gen-identity as the identity of the structures of nature and mathematical structures, as a specifically precised correspondence between the universe of mathematical objects and natural objects, as the belonging of mathematical entities to nature, that is as the empirical nature of mathematical entities, as the mathematical ontic nature of nature itself (it would consist of mathematical entities and

2 Different views on the problem of the mathematicalness of nature can be found in the collective work *Matematyczność przyrody*, eds. M. Heller, J. Życiński, A. Michalik, OBI, Kraków 1992[2], with lectures delivered at the symposium *Dlaczego przyroda jest matematyczna? (Why is Nature Mathematical?)*, organized by Centre for Interdisciplinary Studies at the Faculty of Philosophy of the Pontifical Academy of Theology in Kraków.

3 J. Życiński, *Jak rozumieć matematyczność przyrody*, in: *Matematyczność przyrody*, op. cit., 39.

4 M. Heller, *Co to znaczy, że przyroda jest matematyczna?*, in: *Matematyczność przyrody*, op. cit., 14–15.

mathematical structures or objects indistinguishable from mathematical ones)"[5].

Arguments justifying the hypothesis of the mathematicalness of nature[6] can be found in both the history of mathematics and natural sciences, and in research practice of scientists. I will quote three examples to support this hypothesis.

The first example refers to the history of physics. At the turn of 20th century, Max Planck introduced the concept of an elementary quantum of action to propose a formula for radiation of a perfect black body. This was an "almost fabricated"[7] concept, as Grzegorz Białkowski writes. Planck attempted to incorporate this concept into classical physics, but it was, as Planck himself admitted, "stubborn and resistant"[8]. As further advancements in physics have shown, the concept of quantum of action turned out to be extremely prolific and it became the foundation of quantum theory. In this sense, it can be said that it opened up new and unexpected perspectives for physics. The idea of quantum of action has had much more impact than Planck himself expected from it. Planck considered this type of concepts as so-called absolute elements. They are fixed elements of the theory of physics and are preserved even if the entire theory changes[9]. Apart from the quantum of action, Planck considers the laws of conservation of energy, momentum and the principle of minimal action as absolute elements[10]. These absolute elements were for Planck the "signs" of the real physical world, "na-

5 M. Czarnocka, *Matematyczność przyrody w uwikłaniu epistemologicznym*, in: *Nauka w filozofii. Oblicza obecności*, eds. S. Butryn, M. Czarnocka, W. Ługowski, A. Michalska, IFiS PAN, Warszawa 2011, 270.

6 Stanisław Wszołek points to the fact that the thesis about the mathematicalness of nature is a metaphysical hypothesis: S. Wszołek, *Matematyka i metafizyka. Krótki komentarz na temat hipotezy matematyczności świata*, Studia Philosophiae Christianae 46(2010)1, 25–36.

7 G. Białkowski, *Stare i nowe drogi fizyki. Fizyka XX wieku*, Wiedza Powszechna, Warszawa 1982, 24.

8 Ibid.

9 M. Planck, *Nowe drogi poznania fizycznego a filozofia*, ed. S. Butryn, transl. K. Napiórkowski, IFiS PAN, Warszawa 2003, 194, 249.

10 Ibid, 104, 162.

ture ... revealed a certain absolute, a certain actually unchangeable unit"[11]. Such absolute elements are, as demonstrated by Magdalena Filipek, the principles of symmetry which play a significant role in contemporary physics[12]. Absolute elements are identified at the level of the theory of physics. At the same time, they have a relation to natural reality. Thus, it can be concluded that there is a correspondence between the structures of nature and mathematical formulas that capture absolute elements as understood by Planck.

The second example is related to a story told by Olaf Pedersen. As a young physics teacher he taught children about the specific weight of bodies. The "traditional" way of introducing this concept from its definition to experimental determination of the specific weight of metals did not arouse much interest of pupils. So Pedersen came up with the idea to start with measuring the weight and volume of different pieces of lead. Pupils were given two columns of numbers. Then Pedersen suggested that they do something with those numbers. After ineffective attempts to add and multiply the numbers, pupils started dividing them. "And then – a miracle happened – as the result of the operation, each pair of numbers yielded almost the same result. I will never forget the silence which suddenly fell over the classroom", Pedersen writes[13]. Nature revealed one of its properties through a mathematical formula. This experience of pupils can be extended to include the experience of scientists dealing with the usefulness of mathematics in research pertaining to the world. A certain mathematical formula reveals the physical reality, discloses interesting aspects of the physical world.

The third example demonstrates the special connection between the world of physical experiment and mathematics. In maths, so-called

11 Ibid, 181.

12 M. Filipek, *Elementy absolutne w fizyce w kontekście koncepcji trzech światów Maxa Plancka*, in: *Z zagadnień filozofii przyrodoznawstwa i filozofii przyrody*, vol. 20, eds. A. Lemańska, M. Lubański, A. Świeżyński, Wydawnictwo UKSW, Warszawa 2011, 402–433; Idem, *Elementy absolutne w fizyce w kontekście filozofii Maxa Plancka*, Studia Philosophiae Christianae 44(2008)2, 230–237.

13 O. Pedersen, *Wiara chrześcijańska i przemożny urok nauki*, transl. T. Sierotowicz, in: *Stwórca – Wszechświat – Człowiek*, vol. 1, ed. T. Sierotowicz, OBI – Biblos, Tarnów 2006, 78.

quantum algorithms are formulated which can be used for proving mathematical theses by performing a quantum experiment. Such an algorithm is, for example, Shor's algorithm for integer factorisation. If a quantum computer was constructed, this algorithm would enable a quick factorisation of each integer[14]. Thus, traditional mathematical proof can be replaced with physical experiments. The existence of quantum algorithms can then constitute a premise of the argument supporting the relation between mathematical structures and natural reality.

The above examples demonstrate that there are patterns in nature that can be captured with the use of mathematical formulas. But this is an understanding of the mathematicalness in its weakest sense. There is still no explanation why such patterns exist in nature. The mathematicability of nature can be explained with the use of a much stronger hypothesis of the mathematicalness of nature connected with mathematical platonism. Such an extreme version of the mathematicalness of nature is shared by Michał Heller and Józef Życiński. They argue that the foundation of natural reality consists of mathematical structures that are existentially primary in relation to the material world. As Heller notes, "If, for example, two elementary particles collide and produce a cascade of other particles, this happens not because they have some mysterious power and it was just a fortunate coincidence that some mathematical model can aptly … describe this phenomenon, but because these particles are an actualisation of a certain mathematical structure … and they perform exactly what is encoded in that structure. If there was no mathematical structure, there would be no particles"[15]. According to Życiński, it is not concrete things perceived by us, but the relational formal structures that constitute the

14 K. Wójtowicz, *Teoria obliczeń kwantowych – argument w sporze o aprioryczny status matematyki?*, Studia Philosophiae Christianae 45(2009)1, 71–91; Idem, *Empiryczne aspekty dowodów matematycznych*, in: *Światy matematyki. Tworzenie czy odkrywanie?*, eds. I. Bondecka-Krzykowska, J. Pogonowski, Wydawnictwo Naukowe UAM, Poznań 2010, 341–365. It is worth adding that a "typical" algorithm for integer factorisation is extremely time-consuming.

15 M. Heller, *Fizyka i meta-fizyka*, in: *Ponad demarkacją*, eds. W. Kowalski, S. Wszołek, Biblos, Tarnów 2008, 100.

primary level of the physical world[16]; "material particles have dematerialised to become a manifestation of directly unobservable fields whose structure and interactions are described by the mathematical formalism of theory"[17]. Życiński then assumes "the ontic primacy of relations and structures over their physical and biological realisation"[18]. What is hidden behind the concrete objects perceivable with the senses, is the platonic reality that lies at the foundation of physical processes[19]. This platonic reality is defined by Życiński as the "field of rationality". It constituted the "foundation" of natural reality.

3. THE DIFFICULTIES OF THE HYPOTHESIS OF MATHEMATICALNESS OF NATURE

There is a range of arguments supporting the weaker version of the hypothesis about the mathematicalness of nature. But do these arguments support the version adopted by, among others, Heller and Życiński? Are there any data indicating that natural objects are really a realisation of mathematical structures? A prerequisite for applying mathematics is the idealisation or abstraction of a particular fragment of natural reality. Therefore, do mathematical theories used in physics capture the structure of the world, or just our idealised representation of the world? Is mathematics just a useful tool, or do its theories reflect the natural reality? And hence, is nature mathematical, or just mathematicable? The existence of quantum algorithms can be

16 "Along with the advancement of knowledge, the reality of the observed substrate and particles appears to be secondary, and the network of relations and structures described in the language of mathematics seems to be a fundamental and primary reality. These structures can have diverse physical realisations, which does not change the fact that the level of symmetry, invariants and formal relations remains a more primary level of existence" (J. Życiński, *Teizm i filozofia analityczna*, vol. 2, Znak, Kraków 1988, 67).

17 Ibid, 60.

18 M. Heller, J. Życiński, *Wszechświat i filozofia. Szkice z filozofii i historii nauki*, Polskie Towarzystwo Teologiczne, Kraków 1980, 66.

19 J. Życiński, *The rationality field and the laws of nature*, in: *Wyzwania racjonalności. Księdzu Michałowi Hellerowi współpracownicy i uczniowie*, eds. S. Wszołek, R. Janusz, Wydawnictwo WAM – OBI, Kraków 2006, 92.

used to support the mathematicalness of nature in the "weaker" sense, without assuming a platonic perspective. However, mathematicalness of nature remains something mysterious in this approach. The hypothesis of mathematicalness of nature in its extreme version explains why the structures of nature and mathematical structures fit together. However, it is a view that generates more problems than explanations.

3.1. THE PROBLEM OF CHOOSING A MATHEMATICAL THEORY BY A SCIENTIST

A scientist, when formulating a natural-science theory, either perceives that some mathematical theory "fits" to the description of a physical theory, so he chooses it from among the mathematical theories known to him, or formulates a new mathematical formalism, at times without initially sufficient justification in the field of mathematics (as in the case of Dirac delta) and formulates a natural-science theory on its foundation.

It seems that a scientist enjoys a lot of freedom when choosing a mathematical theory. For it happens that the same phenomena can be captured with the use of different mathematical formalisms. This was the case, for example, of formulating the theory of micro-universe. In this case there are different mathematical formalisms, though they are "translatable" one to another. However, it is difficult to determine which of the ontologies of mathematical theories corresponds to the structure of nature. Attempts are also made at developing theories of physics on the basis of mathematical formalisms different than the ones that are generally used in physics, or even eliminating mathematical concepts from the theory of physics[20]. Although these "operations" are performed by philosophers

20 For example, Paweł Zeidler demonstrates the possibilities provided to physics by the so-called alternative set theory or non-standard analysis. These theories determine other "ontologies" of physical theories. P. Zeidler, *Spór o status poznawczy teorii. W obronie antyrealistycznego wizerunku nauki*, Wydawnictwo Naukowe IF UAM, Poznań 1993, 86–103. On the other hand, the best known attempt to eliminate abstract concepts from physics is nominalism (fictionalism) proposed by Hartry Field who attempts to demonstrate that mathematics is not indispensable for physics (in this way

rather than physicists active in the field of developing physics, they nonetheless demonstrate that the choice of a mathematical theory by a scientist is not fully determined. Therefore, does a physicist discover some mathematical structure "embodied" in nature, or does he impose on nature his own conceptual structure enabling him to engage in a dialogue with nature? It seems that there is no clear answer to a question formulated in this way. Undoubtedly, certain phenomena seem to impose a mathematical approach, however this does not apply to all of them.

What is more, if a mathematical theory is to be applied in physics, as a rule, the investigated reality has to be "simplified". For example, in cosmology it is assumed that distribution of matter in the universe is homogeneous, that space is isotropic, that in the entire universe the same laws of physics apply as on Earth. These assumptions make it possible to solve the equations of the general theory of relativity used for the entire universe and construe a cosmological model.

The issue of the selection of a mathematical formalism is to some extent associated with problems concerning the measurement of, and units used for the measurement of a variety of dimensions. On the one hand, it seems that a scientist is completely free to choose the units of measurement. On the other, as pointed out by Grzegorz Białkowski, this choice is determined by the ease of performing calculations and by the possibility for other scientists to verify the results of such measurements[21]. Therefore, some units are

Field attempts to disprove the second assumption of Quine-Putnam's argument for mathematical realism). According to Field, the use of mathematics in physics is motivated by convenience – theories then become simpler. In particular, Field formulates Newton's theory of gravitation as a nominalist theory. Cf. H. Field, *Science without Numbers*, Basil Blackwell, Oxford 1980.

21 "Of course, each researcher could express the results of his measurements in any units, for example measure length with his own feet. However, if this method was applied, the results obtained by him could not be verified by other researchers. What is more, units belonging to such a system as an inch (the width of the human thumb), foot, cubit, mile etc., are in complex arithmetic relations which makes it difficult to effectively apply them. It seems obvious that the decimal metric system is the best choice for the application in some principles adopted in physics which include intersubjective verifiability and the convenience in the use of the calculation apparatus."

more convenient than other. Nonetheless, it is not an argument in favour of the mathematicalness of nature. The choice of the units of measurement is to a great extent conventional.

Before applying a particular mathematical theory, a scientist usually idealises or abstracts the analysed aspects of natural reality. As a consequence, theories of natural science capture the properties of ideal objects, such as a point particle, a perfect gas, a perfect black body that do not exist in natural reality. Newtonian mechanics and the special theory of relativity assume the existence of the inertial reference frame including the entire space. This enabled the formulation of useful theories pertaining to the movement of point particles, despite the fact that such global systems do not exist in nature. However, without this assumption, attempts to formulate a theory of movement yielding accurate predictions ended up in failure[22]. As noticed by Jarosław Mrozek, when analysing Einstein's theory in this scope, we are dealing with a triple relationship: the natural world – theories of physics – mathematics[23]. In this approach, the structures of the natural science are between the structures of nature and the structures of mathematics. Thus, mathematical structures constitute a foundation for idealised abstract models of certain aspects of reality which are the subject matter of theories. But do these models adequately capture the structure of nature? Do they reflect the structure of the world? To provide an affirmative answer to this question, we would have to state that abstraction and idealisation do not oversimplify reality and thus do not "distort" physical structures, which is closely connected with the necessity to adopt a realistic interpretation of natural science theories.

G. Białkowski, *Ciągłość i nieciągłość w fizyce*, Delta (1977)8 (http//www.wiw.pl/delta/ciaglosc.asp), [accessed on: 08/2012].

22 As noted by Jerzy Kowalski-Glikman, "with the use of mathematics, we are able to describe only idealised processes which are simple enough for their mathematical model to be effectively used for obtaining the predictions of the course of such a process" (J. Kowalski-Glikman, *Cena matematyki*, in: *Nauka* w *filozofii. Oblicza obecności*, op. cit., 224).

23 J. Mrozek, *Czy Einstein głosił matematyczność przyrody?*, in: *Nauka* w *filozofii. Oblicza obecności*, op. cit., 266.

A problem which remains unresolved is the question whether a real process affected by unidentifiable factors can be captured mathematically, without abstracting. Difficulties with mathematising complicated processes are particularly evident in the biological sciences which are difficult to mathematise. As noted by Izrael Gelfand, paraphrasing the title of Winger's Article, "unreasonable is the ineffectiveness of mathematics in biology"[24]. In accordance with the thesis of the mathematicalness of nature, mathematical structures correspond with natural structures. It seems, however, that correspondence exists only between the structures appearing in physical models and mathematical structures.

3.2. MATHEMATICALNESS OF NATURE AND THE DETERMINISTIC CHAOS

Problems related to matching mathematical and natural structures are particularly evident in the study of phenomena which involve deterministic chaos. Because, if a phenomenon is really determined and its course is sensitive to the change of initial conditions, then it is practically impossible to distinguish, based on experimental data, the specific function which models a given phenomenon. We can only choose from among the classes of a variety of functions, and this is also done only in an inaccurate way. As Ian Stewart notes, "any theory in the same universality class will do just as well"[25]. Therefore, it is impossible to choose one particular model for describing a phenomenon: models with different parameters, or even completely different models can, within the range of measurement error, model a particular process equally well, or equally improperly. We are also unable to distinguish a situation in which exponentially accumulated measurement errors exist with the model no longer

24 L. Sokołowski, *Parę uwag o matematyczności przyrody*, in: *Nauka w filozofii. Oblicza obecności*, op. cit., 212. The differences between the possibilities to mathematise processes in inorganic and organic nature are also emphasised by M. Czarnocka, *Matematyczność przyrody w uwikłaniu epistemologicznym*, op. cit., 273.

25 I. Stewart, *Czy Bóg gra w kości? Nowa matematyka chaosu*, transl. M. Tempczyk, PWN, Warszawa 1994, 244.

working for this reason from a situation of inadequate model selection, or even inadequate recognition of the phenomenon as occurring in accordance with the deterministic principle.

What is more, some processes can be either approached with the use of deterministic models, or described with the use of statistical methods. Both of the above-mentioned approaches can be equally good for predicting. Sometimes a statistical approach and treating the course of a particular phenomenon as a random phenomenon can be more convenient or mathematically simpler. Thus, the existence of deterministic chaos causes the inability to distinguish between deterministic process (with deterministic chaos) and a random process. The use of mathematics, formulating a mathematical model that would capture the course of a given process does not allow to solve one of the fundamental problems of material reality, namely the issue of its determinateness. Thus, either we are unable to discover the proper mathematical structures lying at the foundations of nature, or such clearly defined structures do not exist. Therefore, as it seems, the discovery of deterministic chaos puts into question the mathematicalness of nature.

3.3. THE PROBLEM OF CONTINUITY AND INFINITY IN NATURE

Another problem is related with the existence in mathematics of certain concepts for which it is impossible to verify whether there is something that corresponds to them in nature. I will consider two of such mathematical concepts: continuity and infinity. In the theories of physics, various mathematical spaces are the "stage" in which events occur. Mathematical analysis, whose use assumes the continuity (completeness) of a given space and time, is a useful tool for investigating different types of changes in these spaces, because defining the concept of a derivative which is crucial for the study of changes is possible for continuous functions defined on complete spaces[26].

26 As noted by G. Białkowski, "Acceleration is a derivative of velocity with respect to time. Derivatives, as it is commonly known, can be calculated only in the area of arguments in which the differentiated function is continuous. This means that we assume, more or less tacitly, that velocity is a continuous function of time. What are the grounds for

I will limit the question about continuity in nature to the case of movement of objects in the physical space. Theories describing movement are Newtonian mechanics, and special and general relativity theories. In these theories, the stages for events are: Euclidean space, Minkowski spacetime, and pseudo-riemannian spacetime respectively. All these spaces are complete – continuous in colloquial language, time is also continuous. But are physical space and time actually continuous? Or is it just the application of mathematical analysis for the study of changes in nature that requires the "continuising" of space and time? Both our common experience and the natural sciences are unable to provide an answer to the question about continuity of space and time. As noted by G. Białkowski, "At the first sight one could claim that we have a direct experience – be it sensory or introspective experience – of the continuity of space and time. ... However, as exemplified by cinema, such a conclusion is not justified since our nervous system itself combines close moments and close points into continuous entities. What is more, research concerning this system (e.g. vision and sight) indicate that it is completely unable to receive or transmit information in a continuous manner. Such an information within a nerve is as if a volley of electrical discharges which is effected only when a stimulus is strong enough. ... Thus, despite the direct experience of continuity we can see that it has nothing to do with what is 'actually' there"[27]. Neither does scientific experience provide any solution to this problem. This is because we do not have adequate measuring equipment to determine whether space and time are actually continuous. Due to measurement errors and the "inertia" of devices, we can only measure "extensive" fragments of space and time. We therefore cannot differentiate between a continuous change and a change occurring step by step in a very short period of time. What is more, as demonstrated by quantum mechanics, our measurements cannot reach below the so-called Planck's threshold. The assumption that time and space are continuous is the condition for the use of mathematical analysis. It is therefore dictated by the type of

this assumption?" (G. Białkowski, *Ciągłość i nieciągłość w fizyce*, op. cit.).

27 Ibid.

mathematical theory used in physics rather than by the discovery of the real nature of time and space. Does therefore complete (continuous) mathematical space capture the character of natural reality, or is it only its approximation enabling the description of certain phenomena?

Physicists use continuous functions, which is, however, related to the mathematical formalism used, and not to the "actual" character of phenomena in nature. Although Białkowski notes that the use of continuous functions finds its justification in the properties of nature since "what guarantees the continuity of velocity in the theoretical apparatus of physics" is the inertia of matter which is a certain resistance of matter "against changes to its state". "It therefore seems that in the matter itself there are 'continuising' mechanisms which prevent stepwise changes in certain physical dimensions"[28]. Nonetheless, the problem of continuity of space, time and changes occurring in nature still exists. The use of a mathematical formalism in which continuity is assumed does not prove that it also applies to the essence of natural reality. Is therefore the elementary level of the world constructed of mathematical structures, or do we have no other choice but to approximate the real structure of nature with their use.

In research concerning the properties of time and space the question that is asked is not only about their continuity, but also about the related possibility of dividing space and time into increasingly smaller bits. In this context, another concept significant for mathematics appears, which is infinity. And again, the questions that can be asked are: can sections of space and time be divided (even potentially) to infinity, are there any infinite dimensions in nature, is the Universe infinite in space or time, can certain activities be performed an infinite number of times, are time and space composed of an infinite number of points, is there an activity that can be performed in a single moment? Attempts to answer these questions have led to a number of paradoxes. It was already in the antiquity that Zeno of Elea formulated several aporias in which infinity appears in the context of the character of the *continuum*, and from his time, many various paradoxes concerning in-

28 Ibid.

finity have been formulated. It is worth emphasising that there are no simple solutions to these paradoxes to explain all the doubts. Paradoxes therefore show that infinity causes problems. This led to claiming that infinity, especially actual infinity, is a contradictory concept. The situation changed with the development of set theory and, in the 20th century, actual infinity found its place in mathematics.

But can infinity be discovered in nature? Common knowledge allows, at best, for the experience of infinity in its potential sense. Neither does scientific experience give the possibility of direct perception of an actually infinite thing. When we carry our measurements, they are always measurements of finite values of parameters - we do not have adequate tools to measure an infinite dimension. However, it is worth adding that animate nature "invented" potential infinity. The duplication of structures, for example of the DNA, and the reproduction of organisms potentially extend life to infinity, provided that there are inexhaustible resources of energy in nature.

Does therefore infinity exist in nature when what we experience is finite; and even potential infinity seems an abstraction from what is finite, albeit very large and practically unattainable for us? Our common and scientific knowledge do not allow us to answer this question. What is the relevance of the above for the issue of mathematicalness of nature? On the one hand, the assumption about continuity of space and time, and their related infinite divisibility, is necessary for the use of mathematical theories (especially differential and integral equations) for describing some natural phenomena. On the other hand, it seems that actual infinity does not exist in nature and, in any case, it cannot be ascertained. What is more, infinities proposed in the theories of physics are problematic to physicists because it is usually difficult to interpret them from a physical perspective.

For example, cosmology has difficulties with infinity. In the so-called standard model of cosmology, a singularity appears in which the density of matter, pressure and temperature have infinite values, which makes no sense from the point of view of physics. According to this model, the (observable) Universe is limited as to time and space, but it "begins" from a singularity about which the theories of physics have nothing to say. Therefore, the efforts of cosmologists are aimed at removing infinity, especially the infinities pertaining to physical parameters, from the model of the Universe. This is because they are a symptom of a crisis of the theory. Attempts are being made at combining the theory of gravitation with quantum theory because it would enable the description of the initial singularity. But in a variety of formulated concepts, the existence of infinity is also assumed, for example the existence of an infinite number of universes, the eternity of some substrate from which our Universe emerged, the eternal existence of quantum vacuum etc., although the existence of these infinities cannot be proven.

Infinities also appear in quantum theory, for example the infinite-dimensional Hilbert spaces, the theory of which constitutes the foundation for the mathematical formalism of this theory. The model of atom also involves infinity. The idea of the quantisation of energy used for atom leads to a model in which an electron can be simultaneously in an infinite number of places and at an infinite number of energetic levels.

"Inconvenient" infinities also appear in quantum field theories – quantum electrodynamics and quantum chromodynamics. In order to get rid of them from theory, a formal "trick" of renormalisation has been used. However, this is an *ad hoc* procedure without any deeper physical justification.

Therefore, some tension appears between our cognitive possibilities and the theoretical models in which infinities exist. This is why physicists are not fond of infinity. At the same time, infinity naturally appears with the mathematical apparatus. Mathematicians nowadays do not avoid infinity, it can be said that, in a way, they have tamed it. Thus, the situation is that infinity (pertaining to a variety of aspects of nature) is necessary for applying mathematics to the study of nature; at the same time, the demonstration of its existence in nature involves difficulties which have been insurmountable so far. Some infinities are no so much assumed by mathematical formalism, as they appear in solutions to equations in theory. This type of infinities generally cause problems, as it is in the case of the singularity in the model of cosmology. As what a state of "matter" it could be to have an infinite density and temperature?

It seems that potential infinities could be tolerated in nature, and that the existence of actual infinity is an open issue, and a problem from the domain of philosophy rather than the natural sciences: actual infinity cannot be observed, and its occurrence in theory causes problems.

Since there are justified doubts as to continuity of space and time, and as to the existence of infinity in nature, is there actually any correspondence between the structures of nature and structures of mathematics? It is worth adding that when considering these issues one should realise that infinity can appear at two levels: in theories and models, that is in our human theoretical constructs, and in the physical reality whose existence does not depend on us and which we try to understand by formulating scientific theories. If infinity appears in a model, it does not have to automatically mean that such infinities – of space, time, matter, temperature, density, etc. – exist in the Universe as well.

4. CONCLUSIONS

As it seems, the thesis about the mathematicalness of nature is an ontological assumption pertaining to the character of the natural reality and does not stem from the very fact of the application of mathematics in physics. To justify this assumption, it would have to be proven that mathematics captures not just an idealised representation of the natural world, but the actual structure of the world both in the macro- and the micro-scale, and that there is correspondence between natural and mathematical structures. However, it is impossible to demonstrate. The mathematicalness of nature explains the effectiveness of the use of mathematical theories in physics, but the hypothesis itself gives rise to new problems. What is more, adopting the hypothesis of the mathematicalness of nature is connected with ontological assumptions as to the nature of mathematics and the theory of physics. These assumptions also raise a number of objections. Undoubtedly, nature is mathematicable and idealisable, but this does not have to mean that it is mathematical. Thus, the effectiveness of mathematics in the study of nature is a problem which is yet to be solved.

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DOI: 10.21697/spch.2020.56.S2.03